Tutorial 3 MD308/MI 308

(1) Find the radius of convergence of the following power series
  (a) $\sum_{n=0}^{\infty}(x+6)^n$.
  (b) $\sum_{n=0}^{\infty}\frac{(x-3)^n}{10^n}$.
  (c) $\sum_{n=0}^{\infty}\frac{x^n}{10^n n\sqrt{n}}$.
  (d) $\sum_{n=0}^{\infty}\frac{nx^n}{4^n(n^2+1)}$.
  (e) $\sum_{n=0}^{\infty}x^n\ln x$.
  (f) $\sum_{n=0}^{\infty}(x+6)^n$.
  (g) $\sum_{n=0}^{\infty}\frac{x^n}{n(\ln x)^2}$.

(2) Find the singular point of the following ODE and specify the singularity.
  (a) $x^2y''+(x+x^3)y'-2y=0$.
  (b) $x^2y''+(\sin x)y'-2(\cos x)y=0$.
  (c) $x^2y''+(e^x+x^{1/3})y'+(x^2-n^2)y=0$.
  (d) $x^2y''+(x+x^3)y'-2y=0$.
  (e) $y''+(\ln|x|)y'+xy=0$.
  (f) $(x-2)\frac{d^2y}{dx^2}+(\cot\pi x)\frac{dy}{dx}+(\csc^2\pi x)y=0$.
  (g) $x(1+x^2)y''+(\cos x)y'+(1-3x+x^2)y=0$.
  (h) $xy''+e^xy'+(\cos x)y=0$.
  (i) $(x\sin x)y''+3y'+3xy=0$.

(3) Let $y=\phi(x)$ be a bounded solution of the equation $(1-x^2)y''-2xy'+30y=0$. Find the value of $\int_{-1}^{1}(1+x^3+x^5)\phi(x)dx$.

(4) Find the indicial equation for $x(1+x^2)y''+(\cos x)y'+(1-3x+x^2)y=0$.

(5) Find the power series solution of the following differential equations
  (a) $x(x+1)y'-(2x+1)y=0$.
  (b) $(x+1)y'+2xy=0$.
  (c) $(1+x^2)y''-4y=0$.
  (d) $(4+x^2)y''-6xy'+8y=0$.
  (e) $y''+xy'-y=0$.
  (f) $(2+x^2)y''-2xy'+3y=0,\ y(1)=1,\ y'(1)=-1$.

(6) Find the power series solution of the following differential equations
  (a) $2xy''+y'+xy=0$.
  (b) $2x^2y''+xy'+x^2y=0$.
  (c) $2x^2y''+xy'+(x^2-1)y=0$.
  (d) $x^2y''+(x^2+1/4)y=0$.

(7) Find the power series solution of the following differential equations
  (a) $(1-x^2)y''-xy'+p^2y=0,\ p\in\mathbb{R}$ near the point $x=\pm1$(Chebyshev equation).
  (b) $xy''+(1-x)y'+py=0,\ p\in\mathbb{R}$ near the point $x=0$(Lagurre equation).
  (c) $(1-x^2)y''-2xy'+p(p+1)y=0,\ p\in\mathbb{R}$ near the point $x=\pm1$ (Legendre eqution).

(8) Determine whether the point at infinity is an ordinary point, a regular point or an irregular singular point.
  (a) $x^2y''+xy'+2y=0$.
  (b) $y''-2xy'+py=0$.
  (c) $x^2y''+xy'+(x^2-p^2)y=0$.
  (d) $(1-x^2)y''-2xy'+p(p+1)y=0$.
  (e) $y''-xy=0$.

(9) Solve the Bessel's equation $x^2y''+xy'+(x^2-p^2)y=0$ for $p=0,\frac{1}{2}$.

(10) Show that the Bessel's equation $x^2y''+xy'+(x^2-\frac{1}{4})y=0$ can be transformed into $v''+v=0$ by the change of variable $y=x^{-1/2}v(x)$. Conclude that $y_1=x^{-1/2}\cos x$ and $y_2=x^{-1/2}\sin x$ are solutions of the Bessel's equation of order $\frac{1}{2}$.